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NATURAL-CONVECTION HEAT TRANSFER FROM A PLATE WITH ARBITRARY INCLINATION

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Abstract—An experimental study is described concerning natural-convection heat transfer from a plate with arbitrary inclination. The heat is transferred from one side surface of two plates of 30 cm height, 15 cm width and 5 cm height, 10 cm width. The main flow in the boundary layer is restricted two-dimensionally.

The results of heat-transfer coefficients are represented in the relation of average *Nu* number vs. *Ra* number. In the laminar region the expression for the vertical plate is applicable to the inclined plate if only the gravitational term in the *Ra* number is altered to the component parallel to the inclined surface. For the horizontal heated plate and the slightly inclined heated plate with the horizontal both facing downwards, *Nu* number is proportional to one-fifth power of *Ra* number. For the horizontal heated plate facing upwards, the flow in the boundary layer is turbulent and the *Nu* expression agrees with that in the turbulent region for the vertical plate, though the *Nu* number for the smaller plate of 5 cm height is somewhat larger than that for the larger plate of 30 cm height. For the inclined heated plate facing upwards, the larger the angle of inclination becomes, the smaller the transition *Ra* number becomes, and the *Nu* number in the turbulent region agrees with that of the horizontal heated plate facing upwards. The cause of the variation of heat-transfer coefficients with the inclination is explained by the change of flow pattern in the boundary layer shown in photos.

NOMENCLATURE

- g, gravitational acceleration [m/s^2];
- Gr, average Grashof number defined by formula (3);
- Gr_c, Grashof number corresponding to the transition region from laminar to turbulent flow;
- L, length of a heated plate [m];
- Nu, average Nusselt number defined by formula (2);
- Pr, Prandtl number;
- q, average wall heat flux [W/m^2];
- Ra, Rayleigh number, $GrPr$;
- t_w, wall temperature [°C];
- t_∞, fluid temperature [°C].

Greek symbols

- α, average heat-transfer coefficient defined by formula (1) [W/m^2 °K];
- β, average volumetric thermal expansion coefficient [1/°K];
- θ, angle of inclination of a plate with the vertical, plus and minus sign denotes facing downwards and upwards respectively [deg];
- λ, thermal conductivity [W/m °K];
- ν, kinematic viscosity [m^2/s].

1. INTRODUCTION

ABOUT natural convection along an inclined heated plate there are some works published. It is, however, impossible to evaluate precisely heat-transfer coefficients for the arbitrary angle of inclination.

Rich [1] conjectured theoretically that the heat-transfer coefficient for an inclined heated plate could be treated just as for a vertical plate, if the gravitational term in Gr number is altered to the component parallel to the inclined surface. In his experimental data, however, no influence of the angle on the heat-transfer coefficient can be found even qualitatively in the range of inclination angles between 0° and -40°. Michiyoshi [2] solved pertinent boundary-layer equations approximately by substituting a flat elliptical cylinder for a strip. His result exhibits that the heat-transfer coefficient becomes larger than Rich's as the angle of inclination with the vertical becomes large. Kierkus [3] obtained a perturbation solution of boundary-layer equations for an inclined heated plate of a finite length. His result shows that the first term of temperature perturbation gives little influence upon the temperature field, that is, Rich's conjecture is applicable to the heat-transfer coefficient, but the first term of velocity perturbation is added so as to agree better with the velocity field measured. This solution has such an inconvenience that the average heat-transfer coefficient becomes infinite if the second terms of perturbation are superposed. Therefore, the limitation in which the perturbation parameter can be applied is uncertain.

Yamagata [4] proposed a new solution for a horizontal heated plate facing downwards, summarizing previous studies critically. Clifton and Chapman [5] and Singh *et al.* [6] treated with the same problem and gave solutions of similar shape with slightly different coefficients. These results have not been compared with experiments of two-dimensional flow. It is certain that the characteristic of the heat transfer from a horizontal heated plate facing downwards is not a limit case of that from an inclined heated plate. The angle of inclination bounding between these characteristics is uncertain yet.

Vliet [7] performed an experiment of the heat transfer from an inclined heated plate facing upwards of uniform heat flux to water of uniform temperature. The main flow in the boundary layer was restricted two-dimensionally, the maximum angle of inclination was -60° with the vertical, and the distributions of local heat-transfer coefficients were measured. From the data measured Vliet found that Rich's conjecture held good in the laminar region and there was no influence of the angle of inclination in the turbulent region. These conclusions agree basically with previous experimental results of the authors [8]. Vliet found also the transition range from laminar to turbulent region.

The subject of this paper is to represent the change of characteristics of both the heat-transfer coefficient and the flow pattern with the angle of inclination. There are used two heated plates of 30 cm height, 15 cm width and 5 cm height, 10 cm width, from one side surface of which heat is transferred to water and the angle of inclination of which is able to be taken arbitrary. The main flow in the boundary layer is parallel to the length of the plate. Measurements are performed under such conditions that the temperature stratification in water vessel is being formed and that the plate surface is of neither uniform heat flux nor isotherm. These conditions are most practical. No local heat-transfer coefficients but average ones are measured, and yet these may be useful enough to clarify the influence of the angle of inclination.

2. EXPERIMENTAL APPARATUS AND MEASUREMENTS

Figure 1 shows an experimental apparatus used. A plate made of brass of 30 cm height, 15 cm width and 1 cm thickness is heated by a sheath heater of 2·3 mm dia., which is buried with 6 mm pitch in a solder sheet. For minimizing the backward heat flow from the heater, a counter heater is employed and asbestos of 12 mm thickness is sandwiched between these two heaters. For the sake of water proof, all of these except for the heat transferring surface is covered with German silver plate of 0·4 mm thickness, inside which glass fibre is packed. Transparent acrylic acid resin plates stand along the both side edges of the heated plate so that the inflow from each side edge is prevented and the

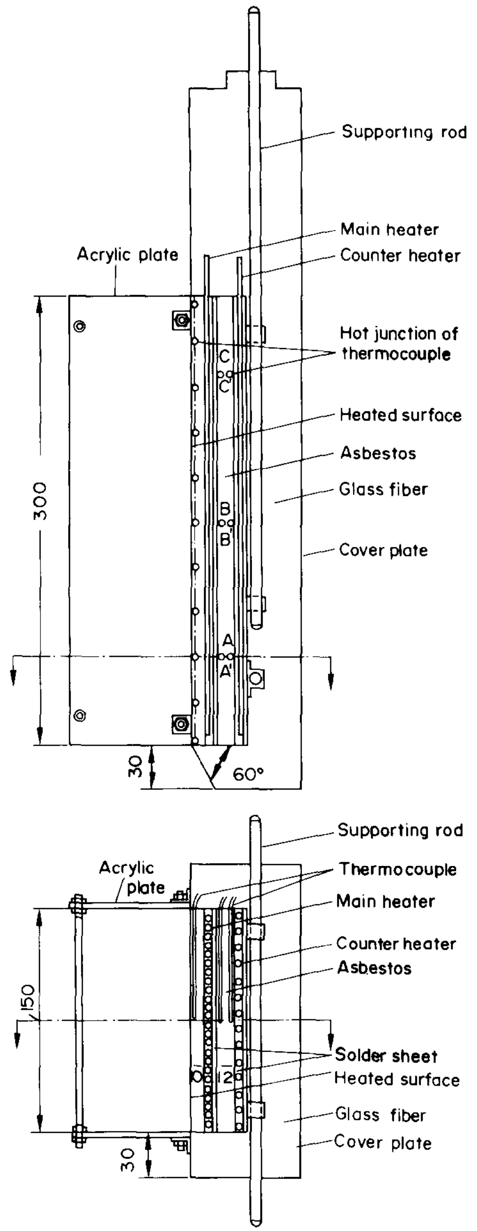


FIG. 1. 30 cm heated plate and its assembly.

Both heated plates were placed in about middle position of cubic vessels of 0·5 × 0·5 m^2 bottom area and 0·6 m depth, 0·35 × 0·35 m^2 bottom area and 0·4 m depth made of transparent acrylic acid resin respectively. The angle of inclination of the heated surface was set by a protractor mounted on a supporting rod and was measured more accurately by a cathetometer. Deaerated pure water was employed for the fluid, the specific resistance of which was more than 2 MΩ cm.

Electric current was supplied to the heaters through variable-voltage transformers, and the electric source was kept at 100·0 ± 0·1 V by a constant-voltage regulator. The power supplied to the main heater was measured by a voltmeter and an ammeter of iron-vain type of 0·5 per cent accuracy.

When the reading of the temperature difference between thermocouples B and B′ fixed on the both surfaces of the asbestos sheet was kept within 0·2°C by regulating the electric input to the counter heater, the backward heat flow from the main heater was estimated to be at most one per cent of the heat input supplied. Heat loss, namely, the heat input to the main heater minus net heat rate transferred from the main flow in the boundary layer is kept two-dimensional. Another heated plate used is made of brass of 5 cm height, 10 cm width and 5 mm thickness, and its assembly is similar to that of 30 cm heated plate.

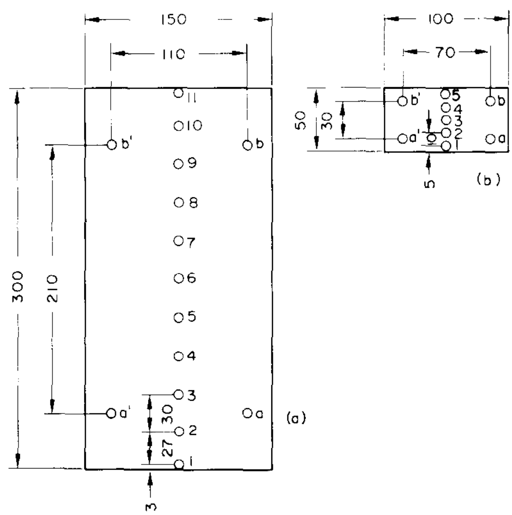


FIG. 2. Positions of the hot junctions of thermocouples. (a) 30 cm heated plate. (b) 5 cm heated plate.

heated surface, therefore, may be confined to the heat transferred from the peripheral edge of the heated plate to the German silver cover. The estimation of the heat loss and its accuracy are described in Appendix.

Fifteen and nine holes of 2 mm dia. were drilled from side edges of the heated plates, and copper–constantan sheath thermocouples of 1·6 mm dia. and copper–constantan thermocouples of 0·3 mm dia. were inserted to each hole for 30 cm and 5 cm heated plate respectively. The positions of hot junctions are shown in Figs. 2(a) and (b) respectively, and the distance from the plate surface to each hot junction is about 2 mm.

For the measurement of temperature stratification of fluid six copper–constantan thermocouples of 0·3 mm dia. were placed at about middle distance between the heated plate and the vessel wall. Each thermocouple was bent for a distance of 5 cm from its hot junction horizontally.

The electro-motive forces of thermocouples were printed on strip-charts of a self-balancing electronic recorder (24 points self-switching, 5 mV full scale). The surface temperature of the heated plate, namely, wall temperature, was obtained by correcting the indication of each thermocouple in proportion to the average heat flux.

3. RESULTS AND CONSIDERATIONS

The experiment was performed in quasi-steady state, which was defined in [9]. The data were obtained after the temperature difference between thermocouples B and B′ was kept within 0·2°K and the temperature difference between the wall and fluid was confirmed to be a constant. To a run it took about 30–40 min and the successive runs of higher heat transfer rate were proceeded. The ranges of experimental data are shown in Table 1, in which the plus

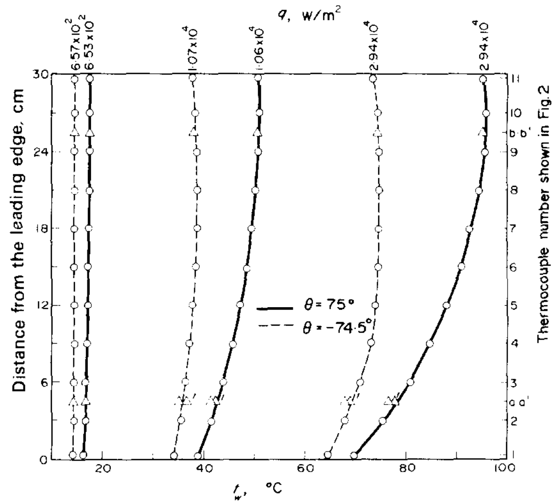


FIG. 3. Examples of the wall temperature distribution of 30 cm heated plate for $\theta = 75°$ and $-74{\cdot}5°$.

Table 1. Ranges of experimental data

	30 cm heated plate				5 cm heated plate			
θ(deg)	90	89	85	80·5	90	89·0	86·8	85·1
	75	45	0	−14·8	79·7	68·6	43·8	0
	−30	−45	−59·9	−74·5	−14·9	−30·0	−44·8	−59·5
	−84·9	−90			−75·0	−79·5	−85·3	−90
q(W/m²)	$1{\cdot}2 \times 10^2 \sim 2{\cdot}8 \times 10^4$				$7{\cdot}8 \times 10^2 \sim 3{\cdot}6 \times 10^4$			
t_w(°C)	14 ~ 92				12 ~ 95			
t_∞(°C)	9 ~ 56				5 ~ 37			
Pr	9 ~ 2				10 ~ 1			
$GrPr$	$2{\cdot}5 \times 10^8 \sim 1{\cdot}1 \times 10^{11}$				$2{\cdot}2 \times 10^6 \sim 5{\cdot}6 \times 10^8$			
$GrPr\cos\theta$	$2{\cdot}6 \times 10^7 \sim 5{\cdot}7 \times 10^{10}$				$1{\cdot}3 \times 10^5 \sim 2{\cdot}7 \times 10^8$			
Nu	40 ~ 480				10 ~ 85			

sign of θ is for the plate facing downwards and the minus for the plate facing upwards.

Some examples of the distribution of wall temperature and the stratification of fluid temperature are shown in Figs. 3 and 4 respectively. For the plate facing downwards the wall temperature rises remarkably with the height though heating is to be uniform. This is caused mainly by the variation of the local heat transfer coefficient and by the stratification of the fluid temperature. In such case some heat flow in the plate takes place so as to make the wall temperature uniform. Consequently, the thermal condition at the wall is some one between uniform heat flux and isotherm. The temperatures at a, b and a′, b′ in Figs. 2(a) and (b) are also shown in Fig. 3 by symbols △ and △′ respectively. The fact that these are generally lower than those at the same height on the centre line suggests that in the heated plate there may be heat flow toward side edges. In addition, the temperature drop near the trailing edge seems to be caused by both the thinning of the boundary layer due to the edge effect and the heat flow to German silver cover. For the inclined plate facing upwards the wall temperature is far more uniform than that for the inclined plate facing downwards, because the boundary layer flow in the former case is susceptible to instability and gives way to turbulence.

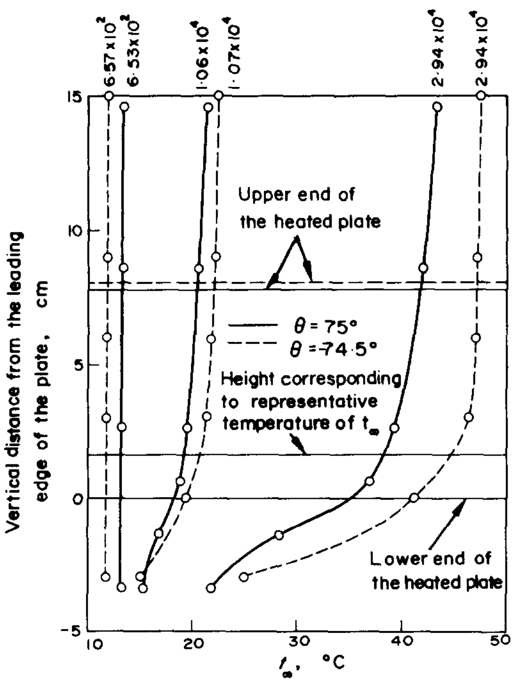


FIG. 4. Examples of the stratification of fluid temperature in the case of 30 cm heated plate for

$\theta = 75°$ and $\theta = -74{\cdot}5°$.

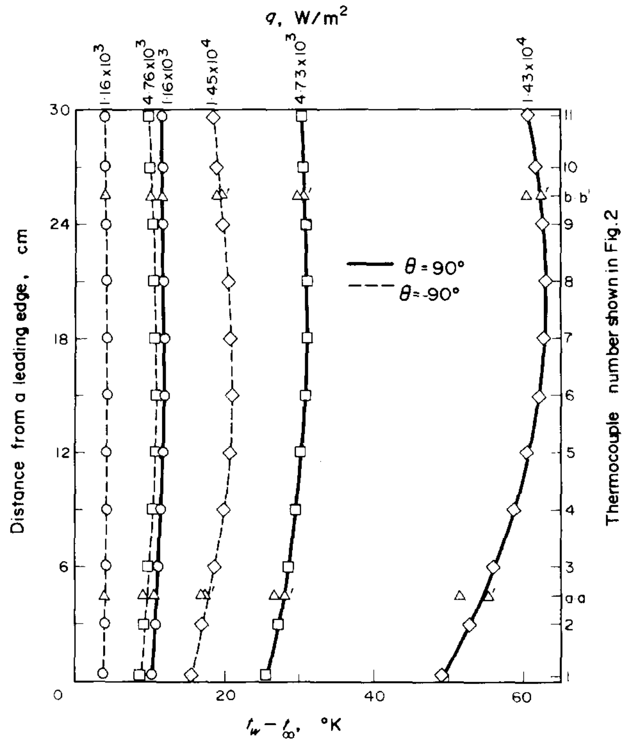


FIG. 5. Examples of $(t_w - t_\infty)$ for the horizontal plate of 30 cm length facing downwards and upwards.

Figure 5 shows some examples of distributions of temperature difference $(t_w - t_\infty)$ in the cases of the horizontal plate facing downwards and upwards. In both cases $(t_w - t_\infty)$ is large near the centre of the plate and small at the edges. In the case of the horizontal plate facing downwards, the boundary layer was entirely laminar. In the case of the horizontal plate facing upwards, on the other hand, it was not laminar over the surface. The fact that the distributions of $(t_w - t_\infty)$ in this case are not symmetrical may be due to unsymmetry of the assembly about the heated plate.

Average heat-transfer coefficient α, average Nusselt number Nu and average Grashof number Gr are defined by the following formulae respectively,

$$\alpha = \frac{q}{t_w - t_\infty}, \qquad Nu = \frac{\alpha L}{\lambda}, \qquad (1), (2)$$

$$Gr = \frac{L^3 g \beta (t_w - t_\infty)}{\nu^2} \qquad (3)$$

where q is average heat flux on the wall. In the cases of the vertical plate and the inclined plate facing downwards, the wall temperature t_w at the middle height was taken as the representative one by referring to [9] and [10]. In the cases of the plate facing upwards and the horizontal plate facing downwards, the mean value of wall temperature was taken as the representative one, since the temperature at the centre of the plate was almost highest as shown in Figs. 3 and 5. The heat-transfer coefficient might be evaluated still somewhat lower than average one, because the representative wall temperature in above cases was estimated by the temperature measurements on the centre line of the heated plate. The fluid temperature t_∞ at the height of $0{\cdot}2L \cos\theta$ from the leading edge of the heated plate was taken as the representative one by referring to [9] and [11]. For volumetric thermal expansion coefficient β the average value from t_∞ to $(t_w + t_\infty)/2$ was used, and for thermal conductivity λ, kinematic viscosity ν and Prandtl number Pr, the values at the temperature $t_e = t_w - 0{\cdot}25(t_w - t_\infty)$ by referring to [9] and [10]. The values of each physical property were referred from Appendix of [9].

Figure 6 shows the relation of Nu vs. $GrPr \cos\theta$ for the vertical plate and the inclined plate facing downwards. Though the boundary layer was not always laminar near the trailing edge for large $GrPr$ values, no influence of the flow regime on the data shown in Fig. 6 is appreciable. The following expression is obtained from the data except for those on 89° inclination angle of 5 cm heated plate,

$$Nu = 0{\cdot}56 (GrPr \cos\theta)^{\frac{1}{4}}, \quad 10^5 < GrPr \cos\theta < 10^{11}. \qquad (4)$$

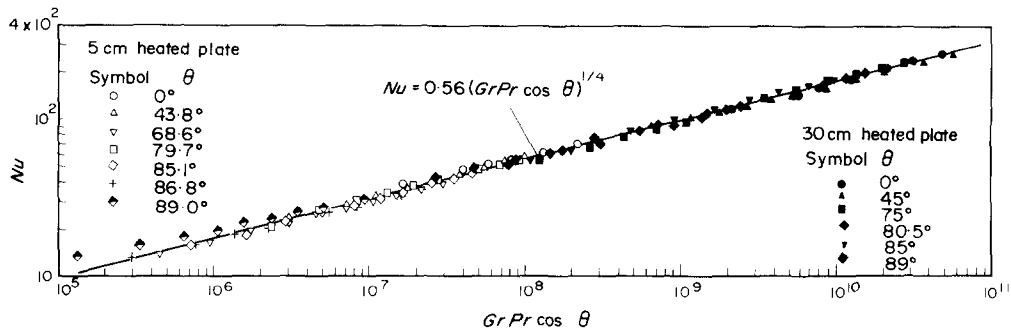


FIG. 6. Relation of Nu vs. $GrPr \cos\theta$ for the vertical plate and the inclined plate facing downwards.

The coefficient of $(GrPr)^{\frac{1}{4}}$ corresponding to the theoretical solution for a vertical plate is about 0·60 for $Pr \approx 5$. The fact that the experimental Nu value is smaller than the theoretical one seems to be caused by estimating the representative wall temperature too high and heat loss too large.

The relation of $Nu/(GrPr)^{\frac{1}{4}}$ vs. θ is shown in Fig. 7, in which the vertical bars indicate ranges of scattering of the data, the solid line expression (4), the chain line Rich's conjecture [1], and the dashed line Michiyoshi's result [2] converted to the case of $Pr = 5$. The experimental

Nu value is about seven per cent smaller than that of Rich's conjecture, however, the influence of inclination angles is identical with that of Rich's. Michiyoshi's solution is larger than Rich's in the range of inclination angle from 75° to 90°. It seems that a more thinner elliptical cylinder must have been employed in substitution for a strip.

One-fifth power of $GrPr$ in expression (5) agrees with theoretical results hitherto reported [4–6] and the coefficient 0·58 is smaller than any one of these theoretical predictions, which are 0·669 for $Pr = \frac{4}{3}$ by Yamagata [4], 0·772 for $Pr = 4{\cdot}2$ by Clifton and Chapman [5] and 0·66 for $Pr \to \infty$ by Singh *et al.* [6]. The value experimentally obtained, however, may be corrected

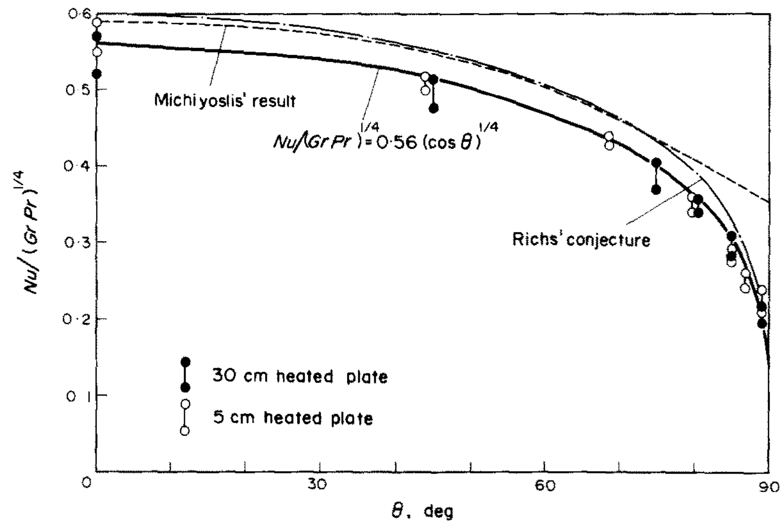


FIG. 7. Comparison of the experimental results with the theoretical ones for the vertical plate and the inclined plate facing downwards.

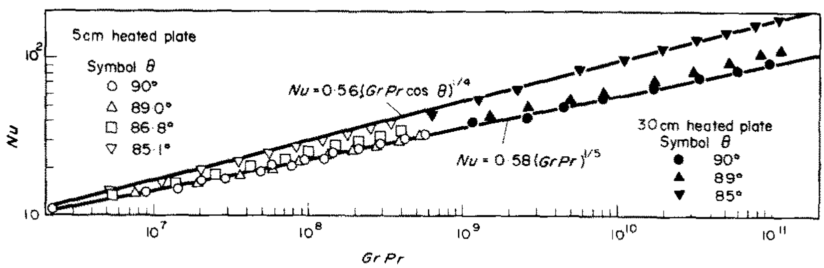


FIG. 8. Relation of Nu vs. $GrPr$ for the horizontal plate and the plate slightly inclined with the horizontal both facing downwards.

Figure 8 shows the relation of Nu vs. $GrPr$ for the horizontal plate facing downwards. The boundary-layer flow was laminar. The following expression is obtained,

$$Nu = 0{\cdot}58(GrPr)^{\frac{1}{5}}, \quad 10^6 < GrPr < 10^{11}. \quad (5)$$

somewhat larger, since Nu number obtained for a vertical plate was about seven per cent smaller than theoretical one, or the length of the heated plate may have to be taken somewhat longer apparently.

In Fig. 8 some data on a plate slightly inclined with the horizontal are also plotted for reference. In the range of $GrPr < 10^9$ the data on 89° with the vertical agree with expression (5), and those on 86·8° somewhat differ from expression (5). In the range of $GrPr > 10^9$ the data on 89° somewhat differ from expression (5). In the whole range of $GrPr$ the data on 85° agree with expression (4). It seems that the range of the inclination angle with the horizontal, within which expression (5) may be applied, becomes smaller with increase of $GrPr$.

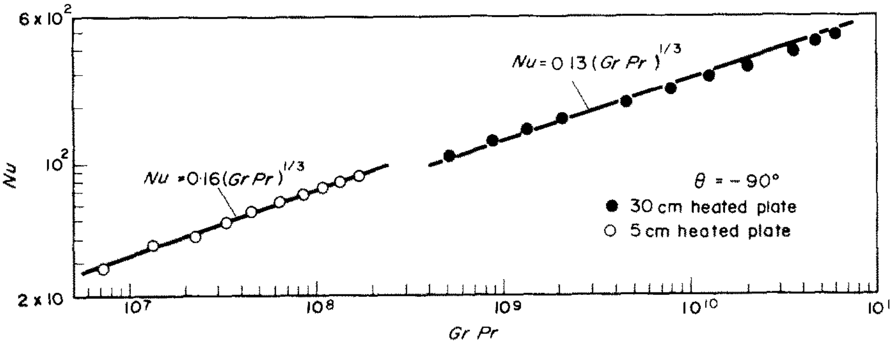


FIG. 9. Relation of Nu vs. $GrPr$ for the horizontal plate facing upwards.

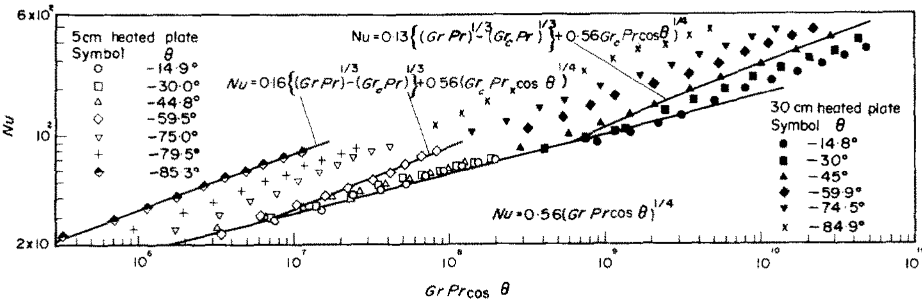


FIG. 10. Relation of Nu vs. $GrPr\cos\theta$ for the inclined plate facing upwards.

Figure 9 shows the relation of Nu vs. $GrPr$ for the horizontal plate facing upwards. In the boundary layer there was no laminar flow. The following expressions are obtained

$$Nu = 0.13(GrPr)^{1/3}, \quad 5 \times 10^8 < GrPr \qquad (6)$$

$$Nu = 0.16(GrPr)^{1/3}, \quad GrPr < 2 \times 10^8 \qquad (7)$$

for 30 cm and 5 cm heated plate respectively. One-third power of $GrPr$ in expressions (6) and (7) suggests that the heat-transfer coefficient does not change locally, consequently that the average heat-transfer coefficient is not affected by the length of the heated plate. The fact that the coefficient in expression (6) for 5 cm heated plate is larger than that in expression (7) for 30 cm plate, however, shows that the above suggestion does not strictly hold but that the heat-transfer coefficients may be large at both edges where the boundary layer starts to develop. For a much longer heated plate the edge effect may become negligibly small. It is remarkable that the heat-transfer coefficient for 30 cm plate agrees with that in the turbulent region along a vertical plate [9].

Figures 10 and 11 show the relations of Nu

vs. $GrPr \cos\theta$ and Nu vs. $GrPr$ for the inclined plate facing upwards respectively. In the laminar region corresponding to the comparatively small angle of inclination with the vertical and comparatively small $GrPr$ number, the same characteristic as expression (4) is recognized, though the scattering of the data is somewhat wider and the Nu number tends to be a few per cent larger than that for the inclined plate facing downwards. These data in the laminar region are omitted in Fig. 11. The expressions (6) and (7) are shown as dashed lines in Fig. 11. The data of the inclination angle between $-60°$ and $-90°$ for 30 cm plate agree well with expression (6).

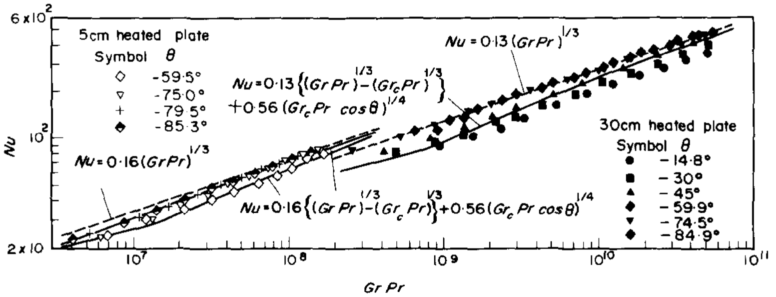


FIG. 11. Relation of Nu vs. $GrPr$ for the inclined plate facing upwards.

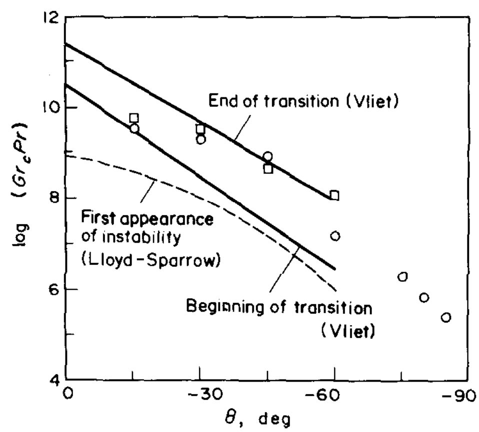


FIG. 12. Relation of $Gr_c Pr$ vs. θ.
○; values of $GrPr$ where Nu starts to separate from the characteristic of the laminar region.
□; values of $GrPr$ where the separation of the boundary-layer flow takes place.

In Fig. 12 are plotted the values of Gr_cPr where the values of Nu in Fig. 10 start to separate from the characteristic of the laminar region and where the separation of the boundary-layer flow was observed respectively. Vliet [7] found modified Rayleigh numbers Gr^*Pr corresponding to the beginning and end of transition indicated in the data of local heat-transfer coefficients for a plate of uniform heat flux. These transition values converted to Gr_cPr are shown as two solid lines in Fig. 12. In spite of different experimental conditions, these data are in fairly good agreement with those of the authors. Lloyd and Sparrow [12] obtained the Rayleigh number corresponding to the first appearance of instability in the laminar boundary layer along an isothermal inclined plate. As shown as a dashed line in Fig. 12, these values are about one order of $GrPr$ lower than those corresponding to the transition for heat-transfer coefficients.

By assuming that expression (4) is applicable in the laminar region of $GrPr < Gr_cPr$ and expression (6) or (7) is applicable in the turbulent region of $GrPr > Gr_cPr$ for an inclined plate facing upwards too, the following expressions are derived,

$$Nu = 0{\cdot}13\,\{(GrPr)^{\frac{1}{3}} - (Gr_cPr)^{\frac{1}{3}}\} + 0{\cdot}56(Gr_cPr\cos\theta)^{\frac{1}{4}} \quad (8)$$

$$Nu = 0{\cdot}16\,\{(GrPr)^{\frac{1}{3}} - (Gr_cPr)^{\frac{1}{3}}\} + 0{\cdot}56(Gr_cPr\cos\theta)^{\frac{1}{4}} \quad (9)$$

for 30 cm and 5 cm heated plate respectively. For examples, the value calculated by expression (8) for $\theta = -45°$ and the values by expression (9) for $\theta = -59{\cdot}5°$ and $-85{\cdot}3°$ are shown as solid lines in Figs. 10 and 11. All of these agree well with corresponding experimental data. By the way, the values of $Gr_c Pr$ at angles between $-75°$ and $-90°$ in Fig. 12 were decided by cut and try adjusting expression (9) to the experimental data for 5 cm heated plate.

Inclusively as for the inclined plate facing upwards, Rich's conjecture is applicable to the heat-transfer coefficient in the laminar region, and after the separation of the boundary-layer flow takes place the heat-transfer coefficient may be considered to be in agreement with those in the turbulent region on a vertical plate. Vliet [7] presented a Nu expression in the turbulent region with the use of a modified Grashof number, however, the heat-transfer coefficients evaluated by the expression have little difference from those evaluated by expression (6).

4. OBSERVATIONS OF FLOW PATTERNS

Flow patterns in the boundary layer are shown in Fig. 13. Figures 13(a) and (b) are mirage photos [9] taken when black lines stretched parallel to the heated plate outside the vessel wall were observed from the opposite side through the boundary layer. Figures 13(c) and (d) are shadow graphs. The flow patterns outside the boundary layer are shown in Fig. 14, the photos in which were taken by suspending aluminium particles and by illuminating them with a light beam through a narrow slit.

The boundary-layer flow along the inclined plate facing downwards was also usually laminar just as shown in Fig. 13(a). In Fig. 13(b) the flow separation takes place at about the centre of the heated plate, and though the down flow after the separation is like laminar the heat-transfer coefficient becomes as high as that in the turbulent region as shown in Figs. 10 and 11. For the horizontal plate facing upwards the upward flow takes place at the centre portion of the plate as shown in Figs. 13(d) and 14(f). With increase of the angle of inclination with the horizontal the position of upward flow is shifted from the centre to the upper edge as shown in Fig. 14(e), and when the angle exceeds about 10° with the horizontal the position of upward flow becomes at the trailing edge of the plate as shown in Figs. 13(c) and 14(d).

Generally, the fluid flows into the boundary layer not perpendicularly to the plate so as the assumption of Kierkus' perturbation solution [3], but almost horizontally. Figures 14(c), and Fig. 14(c)′ which is the schematical sketch of the stream lines, are corresponding to the case of the horizontal plate facing downwards. Most of fluid flows toward the centre portion of the plate horizontally, enters into the boundary layer, and flows toward edges in the boundary layer. Some of fluid flows from the left into the right boundary layer. This unsymmetrical flow seems to be due to the unsymmetry of the assembly about the heated plate.

5. CONCLUSION

(1) For the laminar region of the boundary layer along an inclined plate the average heat-transfer coefficient is expressed as

$$Nu = K(GrPr\cos\theta)^{\frac{1}{4}}$$

where coefficient K may be referred from the theoretical solution for a vertical plate. For the plate facing downwards the applicable range of the angle of inclination in this expression is extended almost to the horizontal, and for the plate facing upwards it is limited by the occurrence of flow separation in the boundary layer, $Gr_c Pr$ at which is shown in Fig. 12.

(2) For the horizontal plate and the slightly inclined plate with the horizontal both facing downwards, the average heat-transfer coefficient is expressed by expression (5). The larger the value of $GrPr$ becomes, the smaller the applicable angle with the horizontal becomes; for instance, about $89° < \theta < 90°$ in the range of

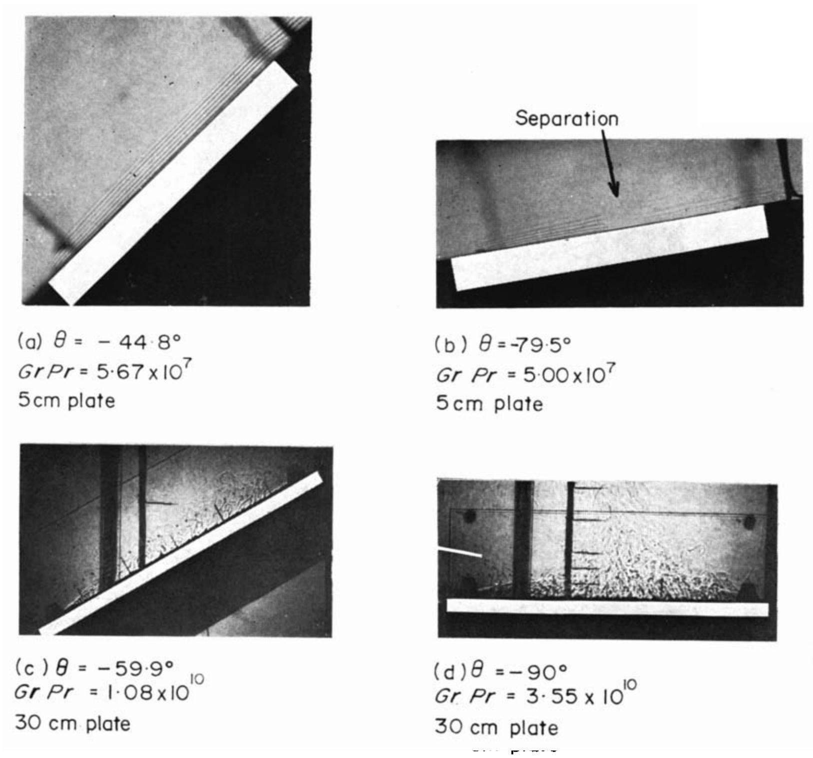
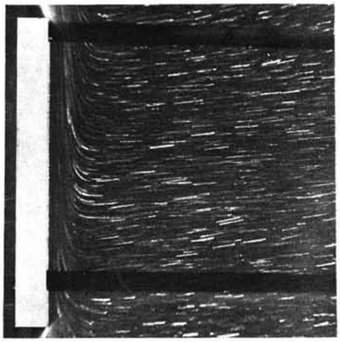
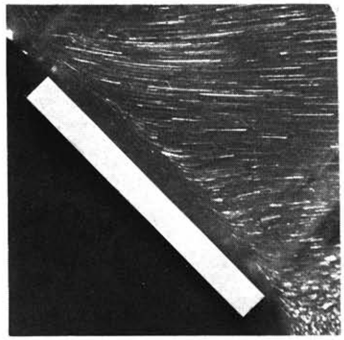


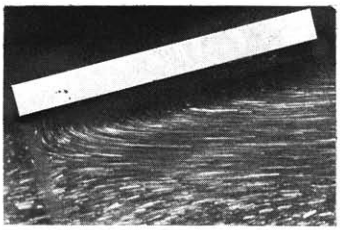
FIG. 13. Flow patterns in the boundary layer.



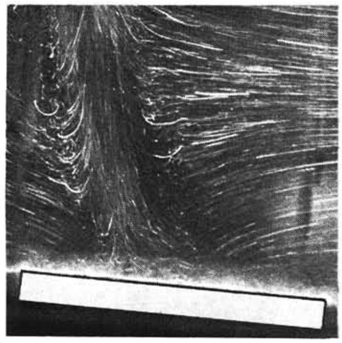
(a) $\theta = 0°$
$Gr\,Pr = 3{\cdot}94 \times 10^7$
5 s exposure



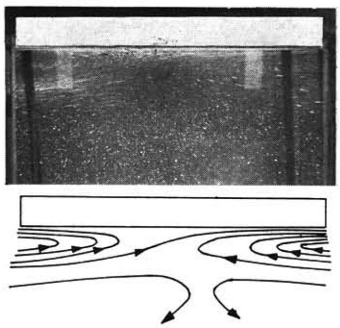
(d) $\theta = -45°$
$Gr\,Pr = 7{\cdot}41 \times 10^7$
5 s exposure



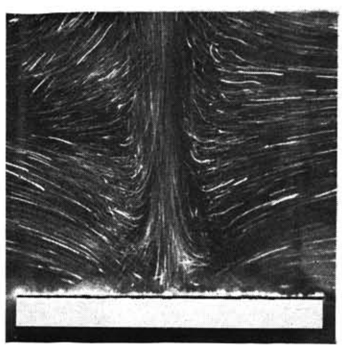
(b) $\theta = 75°$
$Gr\,Pr = 9{\cdot}06 \times 10^7$
10 s exposure



(e) $\theta = -85°$
$Gr\,Pr = 5{\cdot}93 \times 10^7$
5s exposure



(c) $\theta = 90°$
$Gr\,Pr = 2{\cdot}65 \times 10^8$
6 s exposure
(c)' schematical sketch of the streamlines in Fig. (c)



(f) $\theta = -90°$
$Gr\,Pr = 5{\cdot}89 \times 10^7$
5 s exposure

FIG. 14. Flow patterns outside the boundary layer for 5 cm plate.

$GrPr > 10^9$, and about $87° < \theta < 90°$ in the range of $GrPr < 10^9$.

(3) For the horizontal plate facing upwards no laminar flow in the boundary layer can be observed, and the average heat-transfer coefficient is expressed by expressions (6) and (7) for 30 cm and 5 cm heated plate respectively. The heat-transfer coefficient for 30 cm plate agrees with that in turbulent region along a vertical plate, and that for 5 cm plate is about 23 per cent larger than that for 30 cm plate. This difference seems to be due to the effect of the leading edges of the plate.

(4) For the inclined plate facing upwards the heat-transfer coefficient in the region of both turbulent flow and down flow after separation may be considered to be equal to that of the horizontal plate facing upwards. The average heat-transfer coefficient is expressed by expression (8) and (9) for 30 cm and 5 cm plate respectively.

(5) The fluid flows into the boundary layer not perpendicularly to the plate, but almost horizontally. This characteristic is preserved for the horizontal plate too.

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APPENDIX

Estimation of Heat Loss

The heat transferred through the cover plate of German silver from the peripheral edge of the heated plate was considered to be the main heat loss, for which the following approximate calculation was performed.

Four thin plates, which are extended semi-infinitely from each edge of the heated plate, are substituted for the cover plate. By assuming that one side of each plate is thermally insulated and the other side is cooled with a constant heat-transfer coefficient α, an equation of one-dimensional heat conduction is solved. Then the heat rate Q transferred from the total edges of the heated plate, is given by

$$Q = \sqrt{(\alpha\lambda\delta)}\, l\Delta t$$

where λ and δ are thermal conductivity and thickness of the German silver plate respectively, l is total length of the edges of the heated plate, and Δt is temperature difference between the edge of the plate and the ambient fluid.

For α and Δt, the average heat-transfer coefficient and the representative temperature difference measured for respective run were substituted into the above expression. The ratio of the calculated heat loss to the total heat input increased as Δt decreased, for example, 6–19 per cent and 17–34 per cent for 30 cm and 5 cm heated plate respectively. From the comparison between the experimental results for the vertical arrangement and the theoretical ones, it is considered that the heat loss seems to be slightly over-estimated.

The other heat losses through electric lead wires, supporting rods and thermocouples were ignored.

TRANSFERT THERMIQUE PAR CONVECTION NATURELLE POUR UNE PLAQUE A INCLINAISON ARBITRAIRE

Résumé—On décrit une étude expérimentale concernant un transfert thermique par convection naturelle pour une plaque à inclinaison arbitraire. La chaleur est issue d'une face de deux plaques l'une de 30 cm de haut, 15 cm de large et l'autre de 5 cm de haut et 10 cm de large. L'écoulement principal dans la couche limite est bidimensionnel.

Les résultats sur les coefficients de transfert thermique sont traduits par une relation entre le nombre de Nusselt moyen et le nombre de Rayleigh. Dans la région laminaire l'expression pour la plaque verticale est applicable au cas de la plaque inclinée si seul le terme gravitationnel dans le nombre *Ra* est modifié pour faire apparaître la composante parallèle à la surface inclinée. Pour la plaque chauffée horizontale et aussi bien que pour celle légèrement inclinée par rapport à l'horizontale, toutes deux étant orientées vers le bas, le nombre *Nu* est proportionnel à la puissance 1/5 du nombre *Ra*. Pour la plaque chauffée horizontale orientée vers le haut, l'écoulement dans la couche limite est turbulent et l'expression de *Nu* est en accord avec celle dans la région turbulente pour la plaque verticale, bien que le nombre *Nu* pour la plus petite plaque de 5 cm de haut soit plus grand que celui correspondant à la plus grande plaque de 30 cm de haut. Pour la plaque chauffée inclinée et orientée vers le haut, plus grand est l'angle d'inclinaison, plus petit est le nombre *Ra* de transition, et le nombre *Nu* dans la région turbulente est en accord avec celui de la plaque horizontale chauffée et orientée vers le haut. Des photos montrent que la variation des coefficients de transfert thermique en fonction de l'inclinaison est liée au changement de figure d'écoulement dans la couche limite.

WÄRMEÜBERGANG BEI FREIER KONVEKTION AN EINER PLATTE MIT BELIEBIGER NEIGUNG

Zusammenfassung—Es wird eine experimentelle Studie des Wärmeüberganges bei freier Konvektion an einer Platte mit beliebiger Neigung beschrieben.

Die Wärme wird von jeweils einer Oberflächenseite von zwei verschiedenen Platten mit 30 cm Höhe und 15 cm Breite, bzw. 5 cm Höhe und 10 cm Breite übertragen. Der Hauptstrom in der Grenzschicht wird auf Zwei-Dimensionalität beschränkt.

Die Ergebnisse der Wärmeübergangs-Koeffizienten werden in der Relation mittlere Nusselt-Zahl über Rayleigh-Zahl dargestellt. Im laminaren Bereich ist der Ausdruck für die vertikale Platte auch auf die geneigte Platte anwendbar, wenn der Gravitationsterm in der Rayleigh-Zahl entsprechend der zur geneigten Fläche parallelen Komponente geändert wird. Für die horizontale, beheizte Platte und die, gegen die Horizontale leicht geneigte beheizte Platte—beide mit der wärmeabgebenden Fläche nach unten gerichtet—ist die Nusselt-Zahl proportional $Ra^{1/5}$. Bei der nach oben gerichteten, horizontalen, beheizten Platte ist die Grenzschichtströmung turbulent und der Ausdruck für die Nusselt-Zahl stimmt mit dem für den turbulenten Bereich der vertikalen Platte überein, obgleich Nu für die kleinere Platte mit 5 cm Höhe um einiges grösser ist als für die grössere Platte mit 30 cm Höhe. Bei der nach oben gerichteten, geneigten, beheizten Platte wird die Übergangs-Rayleigh-Zahl um so kleiner je grösser der Neigungswinkel wird, die Nusselt-Zahl im turbulenten Bereich stimmt dabei überein mit der Nusselt-Zahl für die nach oben gerichtete, horizontale, beheizte Platte.

Die Ursache für die Änderung der Wärmeübergangs-Koeffizienten mit der Neigung wird mit der Änderung des Strömungsverlaufes in der Grenzschicht anhand von Fotografien erklärt.

ТЕПЛООБМЕН ПРИ СВОБОДНОЙ КОНВЕКЦИИ НА ПЛАСТИНЕ С ПРОИЗВОЛЬНЫМ УГЛОМ НАКЛОНА

Аннотация—Описано экспериментальное исследование теплообмена при свободной конвекции на пластине с произвольным углом наклона. Перенос тепла осуществляется от одной из боковых поверхностей двух пластин высотой 30 см, шириной 15 см и высотой 5 см и шириной 10 см. Основной поток в пограничном слое двумерный.

Результаты измерений коэффициентов теплообмена представлены в виде отношения среднего числа Нуссельта к числу *Ra*. В ламинарном режиме выражение для вертикал-

ьной пластины применимо для наклонной пластины, если только член силы тяжести в числе Ra заменить компонентом, параллельным наклонной поверхности. Для горизонтальной нагреваемой пластины и слегка наклоненной пластины с горизонтальными поверхностями, обращенными в обоих случаях по направлению течения, число Нуссельта пропорционально числу Ra в степени $\frac{1}{5}$. Для горизонтальной нагреваемой пластины, обращенной против течения, течение в пограничном слое является турбулентным, и выражение для числа Нуссельта согласуется с выражением для турбулентного режима в случае вертикальной пластины, хотя число Нуссельта для пластины высотой 5 см несколько больше его значения для пластины больших размеров (высотой 30 см). Для наклонной нагреваемой пластины, обращенной против течения, чем больше угол наклона, тем меньше переходное число Ra, и число Нуссельта в турбулентном режиме согласуется с числом Нуссельта для горизонтальной нагреваемой пластины, обращенной вверх по течению. Это изменение коэффициентов теплообменав зависимости от угла наклона объясняется изменением режима течения в пограничном слое, изображенном на фотографиях.